

# Money Math for Teens

## Break-Even Point



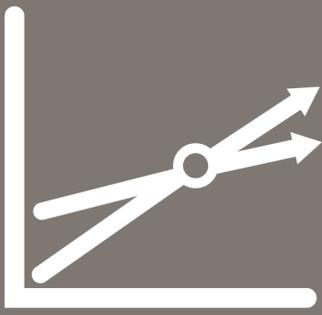


---

*This Money Math for Teens lesson is part of a series created by Generation Money, a multimedia financial literacy initiative of the FINRA Investor Education Foundation, Channel One News and America Saves.*

*Special thanks to Rudy Gawron for preparing the lesson and to Jill Sulam of Transformations Editing LLC for editorial guidance.*

*Money Math for Teens. © Copyright 2014 by the FINRA Investor Education Foundation or FINRA Foundation. Reproduction for nonprofit, educational purposes is permitted and encouraged. All rights reserved.*



# Break-Even Point

## Lesson Plan

---

### OBJECTIVE

Introduce students to fundamental concepts of successfully starting and running a business, focusing on break-even point analysis.

Students will be able to:

- ▶ Understand the vocabulary associated with revenue and expenses
- ▶ Understand the relationship among revenue, expenses and profit
- ▶ Analyze break-even data in a variety of situations
- ▶ Calculate price, sales units, and revenue values for different situations.

### TEACHING MATERIALS

- ▶ Lesson plan
- ▶ Break-Even Point student handout
- ▶ Student assessment worksheet with solutions

### LESSON ACTIVITY

- 1. Determine students' prior knowledge of fundamental vocabulary and concepts by asking questions such as:**
  - What is profit, and where does it come from?
    - It's revenue above and beyond expenses.
  - What is revenue, and how does it differ from income?
    - Revenue is earned income from a business source or occupation. Income can be passive, such as interest and dividends.
  - What is your understanding of a break-even point?
- 2. Present the student handout.**
  - Fundamental equation: Profit = Revenue – Expenses.
  - The **break-even point** is the point where Revenue = Expenses and Profit = 0.
    - Expenses may be fixed or variable.
      - Fixed expenses do not vary based on units sold.
        - They include rent, utilities, insurance and licenses.
        - Labor can be a fixed expense if workers are paid salary or hourly, not by the job.
      - Variable expenses are related to production of each unit.
        - They include raw materials, packaging and per-unit expenses.
        - Labor can be a variable expense if workers are paid per product produced or per service rendered.



## Break-Even Point

- Check students' comprehension so far by working through the example on page 7 of the handout, calculating the break-even point and the profit made from selling 11 instead of 7 units.

Had I sold 11 units instead of 7 units this week, my revenue would have exceeded my expenses and I would have made a profit. How much profit would I have made?

$$P = \text{Price} = 15$$

$$X = \text{Units sold} = 11$$

$$FC = \text{Fixed costs} = 94.50$$

$$V = \text{Variable costs} = 1.50$$

$$Px = FC + Vx + \text{Profit}$$

$$(15)(11) = (94.50) + (1.50)(11) + \text{Profit}$$

$$165 = 94.50 + 16.50 + \text{Profit}$$

$$165 = 111 + \text{Profit}$$

$$\text{Profit} = 165 - 111$$

$$\text{Profit} = \mathbf{\$54.00}$$

- The **contribution margin** is the amount of revenue available to pay the fixed costs after reducing revenue by the variable costs needed to produce the units.
  - Formula: Contribution margin = Revenue – Variable costs
- Check students' comprehension by working through the example on page 8 of the handout.

My product costs \$15, and 25 are sold this week. My variable costs to produce my product are \$1.50 per unit. Can you calculate the contribution margin? Is there a profit?

$$\text{Revenue} = 15 \times 25 = 375$$

$$\text{Variable costs} = 25 \times 1.50 = 37.50$$

$$CM = \text{Revenue} - \text{Variable costs}$$

$$CM = 375 - 37.50 = \mathbf{\$337.50}$$

The example does not say what the fixed costs are, so it can't be determined if the contribution margin is enough to cover the fixed costs and contribute to a profit.

- **Break-even sales units** are the number of units that must be sold to reach the break-even point. Using the break-even point equation,  $Px = Vx + FC$ , you can solve for  $X$  to determine the number of units that need to be sold to break even.

$$Px = Vx + FC$$

$$(Px - Vx) = FC$$

$$x(P - V) = FC$$

$$X = \frac{FC}{(P - V)}$$



## Break-Even Point

Work through the example in the student handout of calculating break-even sales units.

Check students' comprehension with the following exercise:

If I sell a product for \$79.99 that costs me \$11.99 per unit to produce and my fixed costs total \$27,336, how many units must I sell to break even?

$$P = 79.99$$

$$V = 11.99$$

$$FC = 27336$$

$$X = \frac{FC}{(P - V)} = \frac{27336}{(79.99 - 11.99)} = \frac{27336}{68} = 402$$

**X = 402 units**

**Break-even sales dollars** are the amount of revenue needed to reach the break-even point. Once the break-even sales units figure is calculated, then the break-even sales dollars can be determined.

- Formula: Break-even sales dollars = Price per unit x Break-even sales units

$$\text{Break-even sales dollars} = P_X \frac{FC}{(P - V)} = P_X$$

Work through the example in the student handout of calculating break-even sales dollars.

### 3. Lead a class discussion on revenue, expenses and profit. Suggested discussion points and expected responses follow, but student perspectives may lead to interesting conversation.

Think about the formula Revenue = Expenses + Profit. Name three ways to increase profit from a business. Expected responses:

- **Increasing revenue by:**
  - Raising prices. Expenses will remain constant, resulting in an increase in profit.
  - Selling more units. The contribution margin will rise higher than variable expenses, resulting in an increase in profit.
- **Reducing expenses.**
  - Reducing fixed costs, variable costs or both will result in an increase in profit, which will keep the right side of the equation in balance with revenue.
- **Both increasing revenue and reducing expenses** using a combination of the strategies above.



## Break-Even Point

Think about the formula  $\text{Revenue} = \text{Price per unit} \times \text{Number of units sold}$ . Name three ways to increase revenue. Expected responses:

- **Raising prices**
- **Increasing sales**
- **Both of the above.**

Think about the formula  $\text{Expenses} = \text{Variable costs} \times \text{Units} + \text{Fixed costs}$ . Name four ways to reduce expenses. Expected responses:

- **Reduce fixed costs**
- **Reduce variable costs per unit**
- **Reduce sales**
- **All of the above.**

Reducing sales will lower expenses, but is this a conservative business practice that will benefit the company? Explain.

4. **Lead students in a whole-class exercise. The following table can be displayed on a projection screen or written on a board by students with teacher direction.**

Consider Sal and Mario's Pepperoni Delight Restaurant, which only sells pepperoni pizza. The expenses for Sal and Mario's are shown below.

Fixed Costs		Variable Costs Per Pizza	
General Labor	\$1,500	Flour	\$0.50
Rent	\$3,000	Yeast	\$0.05
Insurance	\$200	Water	\$0.01
Advertising	\$500	Cheese	\$3.00
Utilities	\$450	Pepperoni	\$2.00
<b>Total</b>	<b>\$5,650</b>	<b>Total</b>	<b>\$5.56</b>



## Break-Even Point

- A. What is the minimum price Sal and Mario can charge for each pizza?

**\$5.56 covers the variable costs of each pizza, so one pizza could be sold for this price.**

- B. If Sal and Mario price their pizzas at \$10 each, what is their contribution margin?

$CM = \text{Revenue} - \text{Variable cost} = 10 - 5.56 = \mathbf{\$4.44 \text{ per pizza}}$

- C. Calculate the break-even sales units and break-even sales dollar figures.

$$\text{Break-even sales units} = X = \frac{FC}{(P - V)} = \frac{5650}{(10 - 5.56)} = \frac{5650}{4.44} = 1272.5$$

**Break-even sales units: 1,273 pizzas**

Break-even sales dollars = Price per unit x Break-even sales units

**Break-even sales dollars =  $10 \times 1273 = \$12,730$**

5. Evaluate students' comprehension (see assessment worksheet).



## Break-Even Point

### Student Handout: Break-Even Point

So you want to make money? Learning the basics before starting any kind of business is a smart move, because making money requires more than just having a great idea. It's equally important to know where the money comes from and where it all goes.

Throughout this lesson, we will define some important terms, starting with **profit**, which is the amount of revenue you get to keep after all expenses are paid. **Revenue** is the amount of money a business makes. **Expenses** are the costs incurred to make the revenue, and profit is the remaining money after expenses are paid.

If a business sells a product, revenue is the price a product is sold for multiplied by the number of products sold. If a business provides a service instead of selling a product, then the revenue would be the price charged for a service (commonly referred to as a **fee**) multiplied by the number of services sold. If the service is provided to each customer only once in a specific period of time, then revenue is the fee charged multiplied by the number of customers.

There are two types of expenses: fixed costs and variable costs. Fixed costs do not change regardless of how many products are sold or services performed. Examples of fixed costs include rent, insurance premiums and loan payments. Labor costs can be a fixed cost if workers are paid a salary or paid by the hour, since their cost is determined by time and not by how many products they produce.

Fixed costs are usually the same amount each month. Variable costs change depending on how many products are made or how many services are performed. Examples of variable costs include raw materials used to manufacture a product, consumption of fuel during production of the product and packaging of the finished product. If workers are paid by the number of pieces they produce, then labor costs would be a variable cost, too. Variable costs are zero if production is zero. Variable costs will grow as more products are produced. Fixed costs and variable costs add up to total expenses.

If revenue is the money earned, and expenses are paid from the revenue, then profit is the remaining revenue left after expenses:

$$\text{Profit} = \text{Revenue} - \text{Expenses}$$

We can use algebra to reposition the variables for a new formula:

$$\text{Revenue} = \text{Expenses} + \text{Profit}$$

This is actually the same formula, just written in a different way. Notice that the left side of the equal sign shows how much money is earned, and the right side shows where it all goes.



## Break-Even Point

### Break-Even Point (BEP)

In general, the **break-even point**, or **BEP**, is where gains equal losses. In business, the BEP is the point where revenue equals expenses. At this point, there is no profit. For a business, knowing and reaching the BEP is the first major step toward creating a profitable company. The break-even point is when earnings equal the costs to earn them, which means there is no profit and no loss. You break even.

If Revenue = Expenses + Profit, and profit is 0 at the BEP, then Revenue = Expenses at the BEP.

Revenue is the price charged for a product ( $P$ ) multiplied by the number of products sold ( $X$ ) so revenue =  $Px$ .

Expenses are fixed costs + variable costs. Fixed costs are a constant number ( $FC$ ), and variable costs are costs to make each product ( $V$ ) multiplied by the number of products sold ( $X$ ), so expenses =  $FC + Vx$ .

Substituting these values for the big-picture variables in our BEP equation:

$$\text{Revenue} = \text{Expenses at the BEP}$$

$$Px = FC + Vx \text{ at the BEP}$$

### Example >>>

My product costs \$15, and 7 are sold this week. My fixed costs for the week are \$94.50, and my variable costs to produce my product are \$1.50 per unit.

$$P = 15$$

$$X = 7$$

$$FC = 94.50$$

$$V = 1.50$$

$$Px = Vx + FC$$

$$(15) \times (7 \text{ units}) = (1.50) \times (7 \text{ units}) + 94.50$$

$$105 = 10.50 + 94.50$$

$$\mathbf{\$105 = \$105}$$

#### Revenue = Expenses

I broke even this week by selling 7 units at \$15 each. Had I sold 11 units instead of 7 units this week, my revenue would have exceeded my expenses and I would have made a profit. How much profit would I have made?



## Break-Even Point

### Contribution Margin

An important term used in BEP analysis is **contribution margin**. This is the amount of revenue available to pay the fixed costs for a business.

Contribution margin = Revenue – Variable costs

Subtracting the per-unit costs (variable costs) from the revenue earned leaves the contribution margin. If the contribution margin is less than the fixed costs, the BEP has not been reached. When the contribution margin equals fixed costs, you are at the BEP. If the contribution margin exceeds fixed costs, there is a profit.

#### Example >>>

My product costs \$15, and 25 are sold this week. My variable costs to produce my product are \$1.50 per unit. Can you calculate the contribution margin? Is there a profit? Explain your answer.

### Break-Even Sales Units

Using BEP analysis, we can ask and answer some very important questions about a business, such as how many units must be sold to break even.

The answer to this question is called **break-even sales units** and can be calculated as follows.

In the BEP equation,  $Px = Vx + FC$ ,  $X$  is the number of units sold that brings the equation into balance.

$$\begin{aligned} \text{Use algebra to solve for } X: \quad Px &= Vx + FC \\ (Px - Vx) &= FC \\ x(P - V) &= FC \\ X &= \frac{FC}{(P - V)} \end{aligned}$$

Solving for  $X$  shows the number of units that need to be sold to break even.

Calculate the break-even sales units for the following example:

I'm selling a product for \$15 per unit. My variable cost per unit is \$7. My fixed costs are \$9,000. How many units do I have to sell to break even?

**Solution**

$$\begin{aligned} P &= 15 \\ V &= 7 \\ FC &= 9000 \\ X &= \frac{FC}{(P - V)} \\ X &= \frac{9000}{(15 - 7)} \\ X &= \mathbf{1,125 \text{ units}} \end{aligned}$$



## Break-Even Point

### Break-Even Sales Dollars

Another important question can be answered using BEP analysis: How much revenue must I make to break even?

The answer to this question is called **break-even sales dollars** and can be calculated as follows.

In the BEP equation,  $Px = Vx + FC$ ,  $Px$  is the revenue needed to bring the equation into balance,  $P$  is the price of each product, and  $X$  is the number of units necessary to reach the BEP.

Break-even sales dollars = price per unit multiplied by break-even sales units.

Use algebraic substitution to solve one of these equations:

$$\text{Break-even sales dollars} = P \times \frac{FC}{(P - V)} = Px$$

The answer shows the amount of revenue needed to break even.

Calculate the break-even sales dollars for the following example:

I'm selling a product for \$15 per unit. My variable cost per unit is \$7. My fixed costs are \$9,000. What is my break-even sales dollar figure?

#### Solution:

First, calculate the break-even sales units ( $X$ ):

$$P = 15$$

$$V = 7$$

$$FC = 9000$$

$$X = \frac{FC}{(P - V)}$$

$$X = \frac{9000}{(15 - 7)} = \frac{9000}{8}$$

$$X = 1125 \text{ units}$$

Multiply the break-even sales units by price per unit to find the break-even sales dollars.

$$\text{Break-even sales dollars} = 15 \times 1125 = \mathbf{\$16,875 \text{ in revenue.}}$$



<b>Name</b> _____	<b>Date</b> _____
-------------------	-------------------

**Assessment: Break-Even Point**

---

- 1. Blue Corp. shows monthly fixed costs of \$1,797 and per-unit cost of \$9.28. It sells 411 units in a month. What is the minimum price Blue Corp. must sell each unit for to break even?
  
- 2. A start-up company has the following expenses:  
Rent = \$1,100  
Utilities = \$265  
Material and assembly = \$12.65/unit  
Monthly labor = \$625  
If its product sells for \$29.99/unit, how many units must it sell to break even?
  
- 3. I sell a product for \$24.75 and fill orders averaging 37 units per day. My cost to produce and assemble each item is \$3.11. If I am open for business five days per week, what is my contribution margin per unit?
  
- 4. What is the break-even sales dollars figure for an operation that sells 615 products at \$17.50 if each item costs \$7.05 to produce and the fixed costs for the operation are \$3,700/month?
  
- 5. A company sells 900 units/month at \$49.99 each, with an \$18.12 per-unit cost and \$2,175 monthly fixed costs. Is this company making a profit?
  
- 6. Tandem Trucking spends \$11,455 per month to run its business. It makes an average of 420 deliveries per month at a fee of \$55 per delivery. Rounded to the nearest percent, what is Tandem's monthly return on its \$11,455 investment?



Name \_\_\_\_\_

Date \_\_\_\_\_

7. I sell a product for \$35. My fixed costs for the week are \$194, and it costs me \$2.10 per unit to produce the product. If I sell only 6 units this week, how am I doing?
8. Zyleron Corp. shows monthly fixed costs of \$37,210 and a per-unit cost of \$34.79. It sells 275 units in a month. What is the minimum price Zyleron Corp. must sell each unit for to break even?
9. A start-up company has the following expenses:
- |                         |             |
|-------------------------|-------------|
| Rent =                  | \$875       |
| Utilities =             | \$115       |
| Material and assembly = | \$4.75/unit |
| Monthly labor =         | \$480       |
- If its product sells for \$18.99/unit, how many units must it sell to break even?
10. I sell a product for \$21.50 and fill orders averaging 19 units per day. My cost to produce and assemble each item is \$8.47. If I am open for business five days per week, what is my contribution margin each week?



## Break-Even Point

### Assessment Solutions: Break-Even Point

1. Blue Corp. shows monthly fixed costs of \$1,797 and per-unit cost of \$9.28. It sells 411 units in a month. What is the minimum price Blue Corp. must sell each unit for to break even?

$$FC = 1797$$

$$V = 9.28$$

$$X = 411$$

$$Px = FC + Vx$$

$$P(411) = 1797 + 9.28(411) = 411P = 5611.08$$

$$P = \frac{5611.08}{411}$$

$$P = \mathbf{\$13.65}$$

2. A start-up company has the following expenses:

$$\text{Rent} = \$1,100$$

$$\text{Utilities} = \$265$$

$$\text{Material and assembly} = \$12.65/\text{unit}$$

$$\text{Monthly labor} = \$625$$

If its product sells for \$29.99/unit, how many units must it sell to break even?

$$FC = \text{rent} + \text{utilities} + \text{labor} = 1100 + 265 + 625 = 1990$$

$$V = 12.65$$

$$P = 29.99$$

$$X = \frac{FC}{(P - V)}$$

$$X = \frac{1990}{(29.99 - 12.65)} = \frac{1990}{17.34}$$

$$X = 114.76 = \mathbf{115 \text{ units}}$$



## Break-Even Point

3. I sell a product for \$24.75 and fill orders averaging 37 units per day. My cost to produce and assemble each item is \$3.11. If I am open for business five days per week, what is my contribution margin per unit?

$$P = 24.75$$

$$X = 1 \text{ unit}$$

$$V = 3.11$$

Contribution margin = Revenue – Variable cost

$$CM = Px - Vx$$

$$CM = 24.75(1) - 3.11(1)$$

$$CM = \mathbf{\$21.64/unit}$$

4. What is the break-even sales dollar figure for an operation that sells 615 products at \$17.50 if each item costs \$7.05 to produce and the fixed costs for the operation are \$3,700/month?

$$P = 17.50$$

$$V = 7.05$$

$$FC = 3700$$

$$X = \frac{FC}{(P - V)}$$

$$X = \frac{3700}{(17.50 - 7.05)} = \frac{3700}{10.45}$$

$$X = 354 \text{ break-even sales units}$$

$$\text{Break-even sales dollars} = 354 \times \$17.50 = \mathbf{\$6,195}$$

5. A company sells 900 units/month at \$49.99 each, with an \$18.12 per-unit cost and \$2,175 monthly fixed costs. Is this company making a profit?

$$X = 900$$

$$P = 49.99$$

$$V = 18.12$$

$$FC = 2175$$

$$Px = FC + Vx + \text{Profit}$$

$$(49.99) \times 900 = 2175 + (18.12) 900 = 2175 + 16308 + \text{Profit}$$

$$44991 = 18483 + \text{Profit}$$

$$\text{Profit} = 44991 - 18483$$

$$\mathbf{\text{Yes; profit} = \$26,508.}$$



## Break-Even Point

6. Tandem Trucking spends \$11,455 per month to run its business. It makes an average of 420 deliveries per month at a fee of \$55 per delivery. Rounded to the nearest percent, what is Tandem's monthly return on its \$11,455 investment?

$$FC = 11455$$

$$X = 420$$

$$P = 55$$

$$55(420) = 11455 + \text{Profit}$$

$$23100 = 11455 + \text{Profit}$$

$$\text{Profit} = 23100 - 11455$$

$$\text{Profit} = \$11,645$$

$$\text{Return} = \frac{\text{Profit}}{\text{Investment}}$$

$$\text{Return} = \frac{11645}{11455} = 1.0166 = 1.02$$

**Return = 102%**

7. I sell a product for \$35. My fixed costs for the week are \$194, and it costs me \$2.10 per unit to produce the product. If I sell only 6 units this week, how am I doing?

$$P = 35$$

$$FC = 194$$

$$V = 2.10$$

$$X = 6$$

$$Px = FC + Vx + \text{Profit}$$

$$35(6) = 194 + 2.10(6) + \text{Profit}$$

$$210 = 194 + 12.60 + \text{Profit}$$

$$210 = 206.60 + \text{Profit}$$

$$\text{Profit} = 210 - 206.60$$

$$\text{Profit} = \$3.40$$

**I broke even with a slight profit of \$3.40.**



## Break-Even Point

8. Zyleron Corp. shows monthly fixed costs of \$37,210 and a per-unit cost of \$34.79. It sells 275 units in a month. What is the minimum price Zyleron Corp. must sell each unit for to break even?

$$FC = 37210$$

$$V = 34.79$$

$$X = 275$$

$$Px = FC + Vx$$

$$P(275) = 37210 + 34.79(275) = 275P = 46777.25$$

$$P = \frac{46777.25}{275}$$

$$P = \mathbf{\$170.10}$$

9. A start-up company has the following expenses:

$$\text{Rent} = \$875$$

$$\text{Utilities} = \$115$$

$$\text{Material and assembly} = \$4.75/\text{unit}$$

$$\text{Monthly labor} = \$480$$

If its product sells for \$18.99/unit, how many units must it sell to break even?

$$FC = \text{rent} + \text{utilities} + \text{labor} = 875 + 115 + 480 = 1470$$

$$V = 4.75$$

$$P = 18.99$$

$$X = \frac{FC}{(P - V)}$$

$$X = \frac{1470}{(18.99 - 4.75)} = \frac{1470}{14.24} = 103.23$$

$$\mathbf{X = 103 \text{ units}}$$



## Break-Even Point

- 10.** I sell a product for \$21.50 and fill orders averaging 19 units per day. My cost to produce and assemble each item is \$8.47. If I am open for business five days per week, what is my contribution margin each week?

$$P = 21.50$$

$$X = 19 \times 5 = 95/\text{week}$$

$$V = 8.47$$

Contribution margin = Revenue – Variable cost

$$CM = Px - Vx$$

$$CM = (21.50)(95) - 8.47(95)$$

$$CM = 2042.50 - 804.65$$

$$CM = \mathbf{\$1,237.85/\text{week}}$$